

MTH 111

Final Examination

Spring 2012

Question 1. Evaluate the following limits

a)

$$\lim_{x \rightarrow -4^+} \frac{4 + x}{|x^2 + 4x|}$$

b)

$$\lim_{x \rightarrow \infty} \sqrt{x^2 + x + 1} - \sqrt{x^2 - x}$$

c)

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{4x^6 - x}}{3 - x^3}$$

Question 2. Identify the curve represented by the following equation and sketch its graph

$$3x^2 - 2y^2 - 6x + 12y + 3 = 0$$

Question 3.

- a) Write the point $(2, 2)$ in polar coordinates of the form (r, θ) with $r > 0$ and $r < 0$.
- b) Sketch the following polar curve $r = \theta$
- c) Sketch the following polar curve $r(r - 3)(r + 5) = 0$
- d) Sketch the following polar curve $r^2 = 4 \sin(2\theta)$

Question 4. Find $f'(2)$ by using the definition the derivative if $f(x) = \frac{4x}{x-1}$.

Question 5. Find the derivatives of the following functions (do not simplify your answer)

a) Find $f'(x)$ if $f(x) = \frac{\sqrt{x-1}}{(3x-1)^2}$

b) Find $f'(x)$ if $f(x) = e^{\sin(2x)} - \csc(\ln(x^3 - x))$

c) Find $f'(x)$ if $f(x) = \int_{x^2+x}^1 \frac{1}{\sqrt{t-1}} dt$

d) Find the equation of the normal line to $y^2 + 3x - 8y + 3 = 0$ at $(4, 3)$.

Question 6. Find the absolute minimum and the absolute maximum values of

$$f(x) = \frac{x}{x^2 + 1}$$

over the interval $[0, 2]$.

Question 7. Consider the following function $y = \frac{x}{x^2 - 4}$.

- a) Find the domain, the horizontal and the vertical asymptotes,
- b) Find the intervals of increase and decrease,
- c) Find the critical point(s),
- d) Discuss the concavity of the function,
- e) Find the inflection points,
- f) Sketch the graph of $f(x)$.

Hint. $f'(x) = \frac{-(x^2 + 4)}{(x^2 - 4)^2}$ and $f''(x) = \frac{2x(x^2 + 12)}{(x^2 - 4)^3}$.

Question 8. Let \mathcal{R} be the region bounded by the following curves $y = x^2 + 1$ and $y = 4 + 2x$.

a) Sketch the region \mathcal{R}

b) Set up a definite integral that gives the area of the region \mathcal{R}

c) Set up a definite integral that gives the volume of the solid obtained by rotating \mathcal{R} about the line $y = -3$

d) Set up a definite integral that gives the volume of the solid obtained by rotating \mathcal{R} about the line $x = 10$

Question 9. Evaluate the following integrals

a) $\int (e^x + e^{-x})^2 dx$

b) $\int \frac{2x^3 + \sqrt{x} - 1}{x} dx$

c) $\int_{-1}^1 \frac{\sin(x)}{x^2 + 4} dx$

Question 10. A circular plate is being heated. Find the rate of increase in the area of the plate as it expands if the radius is increasing by 3 cm/h and when the radius is 10 cm .

Question 11. Find the maximum slope of a tangent line to the curve $y = 4x^2 + 8x^3$.